

TABLE 1

Some weights for centered approximations at a grid point (generated by setting $M = 4$, $N = 8$, $x_0 = 0$ and $\alpha_\nu = \{0, 1, -1, 2, -2, 3, -3, 4, -4\}$).

Order of derivative	Order of accuracy	Approximations at $x = 0$; x -coordinates at nodes:								
		-4	-3	-2	-1	0	1	2	3	4
0	∞					1				
1	2				$-\frac{1}{2}$	0	$\frac{1}{2}$			
	4			$\frac{1}{12}$	$-\frac{2}{3}$	0	$\frac{2}{3}$	$-\frac{1}{12}$		
	6		$-\frac{1}{60}$	$\frac{3}{20}$	$-\frac{3}{4}$	0	$\frac{3}{4}$	$-\frac{3}{20}$	$\frac{1}{60}$	
	8	$\frac{1}{280}$	$-\frac{4}{105}$	$\frac{1}{5}$	$-\frac{4}{5}$	0	$\frac{4}{5}$	$-\frac{1}{5}$	$\frac{4}{105}$	$-\frac{1}{280}$

O. A.	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	Obser.
2					$-\frac{1}{2}$	0	$\frac{1}{2}$							
4				$\frac{1}{12}$	$-\frac{2}{3}$	0	$\frac{2}{3}$	$-\frac{1}{12}$						
6			$-\frac{1}{60}$	$\frac{3}{20}$	$-\frac{3}{4}$	0	$\frac{3}{4}$	$-\frac{3}{20}$	$\frac{1}{60}$					
8		$\frac{1}{280}$	$-\frac{4}{105}$	$\frac{1}{5}$	$-\frac{4}{5}$	0	$\frac{4}{5}$	$-\frac{1}{5}$	$\frac{4}{105}$	$-\frac{1}{280}$				
10	$-\frac{1}{1260}$	$\frac{5}{504}$	$-\frac{5}{84}$	$\frac{5}{21}$	$-\frac{5}{6}$	0	$\frac{5}{6}$	$-\frac{5}{21}$	$\frac{5}{84}$	$-\frac{5}{504}$	$\frac{1}{1260}$			
12	$\frac{1}{5544}$	$-\frac{1}{385}$	$\frac{1}{56}$	$-\frac{5}{63}$	$\frac{15}{56}$	$-\frac{6}{7}$	0	$\frac{6}{7}$	$-\frac{15}{56}$	$\frac{5}{63}$	$-\frac{1}{56}$	$\frac{1}{385}$	$-\frac{1}{5544}$	

Tabla N° 1. Datos recopilado con la Aplicación Web: Finite Difference Coefficients Calculator

Fuente: <http://web.media.mit.edu/~crtaylor/calculator.html>

TABLE 2

Some weights for centered approximations at a 'half-way' point (generated by setting $M = 4, N = 7, x_0 = 0$ and $\alpha_\nu = \{1/2, -1/2, 3/2, -3/2, 5/2, -5/2, 7/2, -7/2\}$).

Order of derivative	Order of accuracy	Approximations at $x = 0$; x -coordinates at nodes:							
		$-7/2$	$-5/2$	$-3/2$	$-1/2$	$1/2$	$3/2$	$5/2$	$7/2$
1	2				-1	1			
	4			$\frac{1}{24}$	$-\frac{9}{8}$	$\frac{9}{8}$	$-\frac{1}{24}$		
	6		$-\frac{3}{640}$	$\frac{25}{384}$	$-\frac{75}{64}$	$\frac{75}{64}$	$-\frac{25}{384}$	$\frac{3}{640}$	
	8	$\frac{5}{7168}$	$-\frac{49}{5120}$	$\frac{245}{3072}$	$-\frac{1225}{1024}$	$\frac{1225}{1024}$	$-\frac{245}{3072}$	$\frac{49}{5120}$	$-\frac{5}{7168}$

O. A.	$-11/2$	$-9/2$	$-7/2$	$-5/2$	$-3/2$	$-1/2$	$1/2$	$3/2$	$5/2$	$7/2$	$9/2$	$11/2$
2						-1	1					
4					$\frac{1}{24}$	$-\frac{9}{8}$	$\frac{9}{8}$	$-\frac{1}{24}$				
6			$-\frac{3}{640}$	$\frac{25}{384}$	$-\frac{75}{64}$	$\frac{75}{64}$	$-\frac{25}{384}$	$\frac{3}{640}$				
8			$\frac{5}{7168}$	$-\frac{49}{5120}$	$\frac{245}{3072}$	$-\frac{1225}{1024}$	$\frac{1225}{1024}$	$-\frac{245}{3072}$	$\frac{49}{5120}$	$-\frac{5}{7168}$		
10		$-\frac{35}{294912}$	$\frac{405}{229376}$	$-\frac{567}{40960}$	$\frac{735}{8192}$	$-\frac{19845}{16384}$	$\frac{19845}{16384}$	$-\frac{735}{8192}$	$\frac{567}{40960}$	$-\frac{405}{229376}$	$\frac{35}{294912}$	
12	$\frac{63}{2883584}$	$-\frac{847}{2359296}$	$\frac{5445}{1835008}$	$-\frac{22869}{1310720}$	$\frac{12705}{131072}$	$-\frac{160083}{131072}$	$\frac{160083}{131072}$	$-\frac{12705}{131072}$	$\frac{22869}{1310720}$	$-\frac{5445}{1835008}$	$\frac{847}{2359296}$	$-\frac{63}{2883584}$

Tabla N° 2. Datos recopilado con la Aplicación Web: Finite Difference Coefficients Calculator

Fuente: <http://web.media.mit.edu/~crtaylor/calculator.html>

TABLE 3

Some weights for one-sided approximations at a grid point (generated by setting $M = 4, N = 8, x_0 = 0$ and $\alpha_\nu = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$).

Order of derivative	Order of accuracy	Approximations at $x = 0$; x -coordinates at nodes:								
		0	1	2	3	4	5	6	7	8
1	1	-1	1							
	2	$-\frac{3}{2}$	2	$-\frac{1}{2}$						
	3	$-\frac{11}{6}$	3	$-\frac{3}{2}$	$\frac{1}{3}$					
	4	$-\frac{25}{12}$	4	-3	$\frac{4}{3}$	$-\frac{1}{4}$				
	5	$-\frac{137}{60}$	5	-5	$\frac{10}{3}$	$-\frac{5}{4}$	$\frac{1}{5}$			
	6	$-\frac{49}{20}$	6	$-\frac{15}{2}$	$\frac{20}{3}$	$-\frac{15}{4}$	$\frac{6}{5}$	$-\frac{1}{6}$		
	7	$-\frac{363}{140}$	7	$-\frac{21}{2}$	$\frac{35}{3}$	$-\frac{35}{4}$	$\frac{21}{5}$	$-\frac{7}{6}$	$\frac{1}{7}$	
	8	$-\frac{761}{280}$	8	-14	$\frac{56}{3}$	$-\frac{35}{2}$	$\frac{56}{5}$	$-\frac{14}{3}$	$\frac{8}{7}$	$-\frac{1}{8}$

O. A.	0	1	2	3	4	5	6	7	8	9	10	11	12
1	-1	1											
2	$-\frac{3}{2}$	2	$-\frac{1}{2}$										
3	$-\frac{11}{6}$	3	$-\frac{3}{2}$	$\frac{1}{3}$									
4	$-\frac{25}{12}$	4	-3	$\frac{4}{3}$	$-\frac{1}{4}$								
5	$-\frac{137}{60}$	5	-5	$\frac{10}{3}$	$-\frac{5}{4}$	$\frac{1}{5}$							
6	$-\frac{49}{20}$	6	$\frac{15}{2}$	$\frac{20}{3}$	$\frac{15}{4}$	$\frac{5}{6}$	$\frac{1}{6}$						
7	$-\frac{363}{140}$	7	$-\frac{21}{2}$	$\frac{35}{3}$	$-\frac{35}{4}$	$\frac{21}{5}$	$-\frac{7}{6}$	$\frac{1}{7}$					
8	$-\frac{761}{280}$	8	-14	$\frac{56}{3}$	$-\frac{35}{2}$	$\frac{56}{5}$	$-\frac{14}{3}$	$\frac{8}{7}$	$-\frac{1}{8}$				
9	$-\frac{7129}{2520}$	9	-18	28	$-\frac{63}{2}$	$\frac{126}{5}$	-14	$\frac{36}{7}$	$-\frac{9}{8}$	$\frac{1}{9}$			
10	$-\frac{7381}{2520}$	10	$-\frac{45}{2}$	40	$-\frac{105}{2}$	$\frac{252}{5}$	-35	$\frac{120}{7}$	$-\frac{45}{8}$	$\frac{10}{9}$	$-\frac{1}{10}$		
11	$-\frac{83711}{27720}$	11	$\frac{55}{2}$	55	$\frac{165}{2}$	$\frac{462}{5}$	77	$\frac{330}{7}$	$\frac{165}{8}$	$\frac{55}{9}$	$\frac{11}{10}$	$\frac{1}{11}$	
12	$-\frac{86021}{27720}$	12	33	$\frac{220}{3}$	$\frac{495}{4}$	$\frac{792}{5}$	154	$\frac{792}{7}$	$\frac{495}{8}$	$\frac{220}{9}$	$\frac{33}{5}$	$\frac{12}{11}$	$\frac{1}{12}$

Tabla N° 3. Datos recopilado con la Aplicación Web: Finite Difference Coefficients Calculator

Fuente: <http://web.media.mit.edu/~crtaylor/calculator.html>

Cálculo de la Tabla N° 1.

- Para: $-1, 0, 1$

$$\frac{\partial^{(1)}f}{\partial x^{(1)}} \approx \frac{-1f(x-1h) + 0f(x+0h) + 1f(x+1h)}{2h^1}$$

- Para: $-2, -1, 0, 1, 2$

$$\frac{\partial^{(1)}f}{\partial x^{(1)}} \approx \frac{1f(x-2h) - 8f(x-1h) + 0f(x+0h) + 8f(x+1h) - 1f(x+2h)}{12h^1}$$

- Para: $-3, -2, -1, 0, 1, 2, 3$

$$\frac{\partial^{(1)}f}{\partial x^{(1)}} \approx \frac{-1f(x-3h) + 9f(x-2h) - 45f(x-1h) + 0f(x+0h) + 45f(x+1h) - 9f(x+2h) + 1f(x+3h)}{60h^1}$$

- Para: $-4, -3, -2, -1, 0, 1, 2, 3, 4$

$$\begin{aligned} \frac{\partial^{(1)}f}{\partial x^{(1)}} &\approx \frac{3f(x-4h) - 32f(x-3h) + 168f(x-2h) - 672f(x-1h) + 0f(x+0h) + 672f(x+1h) \cdots}{840h^1} \\ &\approx \frac{\cdots - 168f(x+2h) + 32f(x+3h) - 3f(x+4h)}{840h^1} \end{aligned}$$

- Para: $-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$

$$\begin{aligned} \frac{\partial^{(1)}f}{\partial x^{(1)}} &\approx \frac{-2f(x-5h) + 25f(x-4h) - 150f(x-3h) + 600f(x-2h) - 2100f(x-1h) + 0f(x+0h) \cdots}{2520h^1} \\ &\quad \frac{\cdots + 2100f(x+1h) - 600f(x+2h) + 150f(x+3h) - 25f(x+4h) + 2f(x+5h)}{2520h^1} \end{aligned}$$

- Para: $-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6$

$$\begin{aligned} \frac{\partial^{(1)}f}{\partial x^{(1)}} &\approx \frac{5f(x-6h) - 72f(x-5h) + 495f(x-4h) - 2200f(x-3h) + 7425f(x-2h) - 23760f(x-1h) + \cdots}{27720h^1} \\ &\quad \frac{\cdots + 0f(x+0h) + 23760f(x+1h) - 7425f(x+2h) + 2200f(x+3h) - \cdots}{27720h^1} \\ &\quad \frac{\cdots - 495f(x+4h) + 72f(x+5h) - 5f(x+6h)}{27720h^1} \end{aligned}$$

Cálculo de la Tabla N° 2.

- Para $-1/2, 1/2$

$$\frac{\partial^{(1)}f}{\partial x^{(1)}} \approx \frac{-1f(x-0,5h) + 1f(x+0,5h)}{1h^1}$$

- Para $-3/2, -1/2, 1/2, 3/2$

$$\frac{\partial^{(1)}f}{\partial x^{(1)}} \approx \frac{1f(x-1,5h) - 27f(x-0,5h) + 27f(x+0,5h) - 1f(x+1,5h)}{24h^1}$$

- Para $-5/2, -3/2, -1/2, 1/2, 3/2, 5/2$

$$\frac{\partial^{(1)}f}{\partial x^{(1)}} \approx \frac{-9f(x-2,5h) + 125f(x-1,5h) - 2250f(x-0,5h) + 2250f(x+0,5h) - \dots - 125f(x+1,5h) + 9f(x+2,5h)}{1920h^1}$$

- Para $-7/2, -5/2, -3/2, -1/2, 1/2, 3/2, 5/2, 7/2$

$$\frac{\partial^{(1)}f}{\partial x^{(1)}} \approx \frac{75f(x-3,5h) - 1029f(x-2,5h) + 8575f(x-1,5h) - 128625f(x-0,5h) + \dots - \dots + 128625f(x+0,5h) - 8575f(x+1,5h) + 1029f(x+2,5h) - 75f(x+3,5h)}{107520h^1}$$

- Para $-9/2, -7/2, -5/2, -3/2, -1/2, 1/2, 3/2, 5/2, 7/2, 9/2$

$$\frac{\partial^{(1)}f}{\partial x^{(1)}} \approx \frac{-1225f(x-4,5h) + 18225f(x-3,5h) - 142884f(x-2,5h) + 926100f(x-1,5h) - \dots - \dots - 12502350f(x-0,5h) + 12502350f(x+0,5h) - 926100f(x+1,5h) + \dots - \dots + 142884f(x+2,5h) - 18225f(x+3,5h) + 1225f(x+4,5h)}{10321920h^1}$$

- Para $-11/2, -9/2, -7/2, -5/2, -3/2, -1/2, 1/2, 3/2, 5/2, 7/2, 9/2, 11/2$

$$\frac{\partial^{(1)}f}{\partial x^{(1)}} \approx \frac{19845f(x-5,5h) - 326095f(x-4,5h) + 2695275f(x-3,5h) - 15848217f(x-2,5h) + \dots - \dots + 88045650f(x-1,5h) - 1109375190f(x-0,5h) + 1109375190f(x+0,5h) - \dots - \dots - 88045650f(x+1,5h) + 15848217f(x+2,5h) - 2695275f(x+3,5h) + \dots - \dots + 326095f(x+4,5h) - 19845f(x+5,5h)}{908328960h^1}$$

Cálculo de la Tabla N° 3.

- Para 0, 1.

$$\frac{\partial^{(1)} f}{\partial x^{(1)}} \approx \frac{-1f(x+0h) + 1f(x+1h)}{1h^1}$$

- Para 0, 1, 2.

$$\frac{\partial^{(1)} f}{\partial x^{(1)}} \approx \frac{-3f(x+0h) + 4f(x+1h) - 1f(x+2h)}{2h^1}$$

- Para 0, 1, 2, 3.

$$\frac{\partial^{(1)} f}{\partial x^{(1)}} \approx \frac{-11f(x+0h) + 18f(x+1h) - 9f(x+2h) + 2f(x+3h)}{6h^1}$$

- Para 0, 1, 2, 3, 4.

$$\frac{\partial^{(1)} f}{\partial x^{(1)}} \approx \frac{-25f(x+0h) + 48f(x+1h) - 36f(x+2h) + 16f(x+3h) - 3f(x+4h)}{12h^1}$$

- Para 0, 1, 2, 3, 4, 5.

$$\frac{\partial^{(1)} f}{\partial x^{(1)}} \approx \frac{-137f(x+0h) + 300f(x+1h) - 300f(x+2h) + 200f(x+3h) - 75f(x+4h) + 12f(x+5h)}{60h^1}$$

- Para 0, 1, 2, 3, 4, 5, 6.

$$\frac{\partial^{(1)} f}{\partial x^{(1)}} \approx \frac{-147f(x+0h) + 360f(x+1h) - 450f(x+2h) + 400f(x+3h) - 225f(x+4h) + \dots}{60h^1}$$

$$\frac{\dots + 72f(x+5h) - 10f(x+6h)}{60h^1}$$

- Para 0, 1, 2, 3, 4, 5, 7.

$$\frac{\partial^{(1)} f}{\partial x^{(1)}} \approx \frac{-1089f(x+0h) + 2940f(x+1h) - 4410f(x+2h) + 4900f(x+3h) - 3675f(x+4h) + \dots}{420h^1}$$

$$\frac{\dots + 1764f(x+5h) - 490f(x+6h) + 60f(x+7h)}{420h^1}$$

- Para 0, 1, 2, 3, 4, 5, 7, 8.

$$\frac{\partial^{(1)} f}{\partial x^{(1)}} \approx \frac{-2283f(x+0h) + 6720f(x+1h) - 11760f(x+2h) + 15680f(x+3h) - 14700f(x+4h) + \dots}{840h^1}$$

$$\frac{\dots + 9408f(x+5h) - 3920f(x+6h) + 960f(x+7h) - 105f(x+8h)}{840h^1}$$

- Para 0, 1, 2, 3, 4, 5, 7, 8, 9.

$$\begin{aligned} \frac{\partial^{(1)}f}{\partial x^{(1)}} &\approx \frac{-7129f(x+0h) + 22680f(x+1h) - 45360f(x+2h) + 70560f(x+3h) - \dots}{2520h^1} \\ &\quad \frac{\dots - 79380f(x+4h) + 63504f(x+5h) - 35280f(x+6h) + \dots}{2520h^1} \\ &\quad \frac{\dots + 12960f(x+7h) - 2835f(x+8h) + 280f(x+9h)}{2520h^1} \end{aligned}$$

- Para 0, 1, 2, 3, 4, 5, 7, 8, 9, 10.

$$\begin{aligned} \frac{\partial^{(1)}f}{\partial x^{(1)}} &\approx \frac{-7381f(x+0h) + 25200f(x+1h) - 56700f(x+2h) + 100800f(x+3h) - \dots}{2520h^1} \\ &\quad \frac{\dots - 132300f(x+4h) + 127008f(x+5h) - 88200f(x+6h) + 43200f(x+7h) - \dots}{2520h^1} \\ &\quad \frac{\dots - 14175f(x+8h) + 2800f(x+9h) - 252f(x+10h)}{2520h^1} \end{aligned}$$

- Para 0, 1, 2, 3, 4, 5, 7, 8, 9, 10, 11.

$$\begin{aligned} \frac{\partial^{(1)}f}{\partial x^{(1)}} &\approx \frac{-83711f(x+0h) + 304920f(x+1h) - 762300f(x+2h) + 1524600f(x+3h) - \dots}{27720h^1} \\ &\quad \frac{\dots - 2286900f(x+4h) + 2561328f(x+5h) - 2134440f(x+6h) + \dots}{27720h^1} \\ &\quad \frac{\dots + 1306800f(x+7h) - 571725f(x+8h) + 169400f(x+9h) - \dots}{27720h^1} \\ &\quad \frac{\dots - 30492f(x+10h) + 2520f(x+11h)}{27720h^1} \end{aligned}$$

- Para 0, 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12.

$$\begin{aligned} \frac{\partial^{(1)}f}{\partial x^{(1)}} &\approx \frac{-86021f(x+0h) + 332640f(x+1h) - 914760f(x+2h) + 2032800f(x+3h) - \dots}{27720h^1} \\ &\quad \frac{\dots - 3430350f(x+4h) + 4390848f(x+5h) - 4268880f(x+6h) + \dots}{27720h^1} \\ &\quad \frac{\dots + 3136320f(x+7h) - 1715175f(x+8h) + 677600f(x+9h) - \dots}{27720h^1} \\ &\quad \frac{182952f(x+10h) + 30240f(x+11h) - 2310f(x+12h)}{27720h^1} \end{aligned}$$